APPENDIX A

STATEWIDE WATER AND AGRICULTURAL PRODUCTION MODEL

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> *In the world there is nothing more submissive and weak than water.*
> *Yet for attacking that which is hard and strong nothing can surpass it.*
> **Lao-Tzu** -6th century B.C.

INTRODUCTION

The Department of Water Resources Bulletin 160-98 (herein referred to as Bulletin 160-98) forecasts that California's population will increase from 32.1 to 47.5 million people, representing a 48% increase. This increase in population, and environmental water reallocations both contribute to large water distribution imbalances. Historically, water imbalances have been countered by new water supply projects, but increasing environmental concern has effectively eliminated this option. Furthermore, much of California's contemporary water policy is driven by an antiquated allocation mechanism that was originally used to stimulate development in the Western United States. The existing institutional allocation mechanism has permitted divergences between supply and demand. Realigning supply with demand will require a mechanism that is able to *adapt* to stochastic supply events.

The Statewide Water Agricultural Production model (SWAP) is a tool that can be used to illustrate the potential gains from trade by exploiting spatial and temporal differences in the marginal valuation of water. SWAP operates with the objective of maximizing statewide economic returns subject to resource, production, and policy constraints and is able to adapt to the supply conditions (i.e. drought conditions). This model is unique in its ability to identify specific monthly water allocations that exactly match the State's supply to demand by assessing the willingness to pay of different agricultural water users for a *reliable* water supply. The willingness-to-pay for a specific quantity of water can be inferred from- the imputed shadow value on the water to the user. Results from this model can be used in the CALVIN model to estimate the economic valuation of urban and agricultural water demands and how those demands change under various water supply conditions.

Until recently, the use of water markets or water transfers has not been extensively considered as a viable reallocation mechanism to meet the increasing gap between the demand for and the supply of variable surface water. This is largely due to the incorrect assumption that the demand for water is highly price inelastic, suggesting that price changes do not influence quantity demanded. The SWAP model assumes that demands for agricultural water are responsive to changes in crop price and water cost. It is this fundamental assumption that differentiates SWAP's supply-demand approach from the existing models that predict large future water shortages.

SWAP AND CVPM

SWAP extends the research presented in the Central Valley Production Model (CVPM). CVPM was generated as part of the Programmatic Environmental Impact Statement (PEIS) of the Central Valley Project Improvement Act (CVPIA), in order to examine the economic implications on agricultural production as a result of the Central Valley Improvement Act of 1992. This legislation authorized the annual reallocation of 800,000 acre-feet of water away from agricultural uses for environmental enhancement purposes. SWAP uses much of the original data from the CVPM, including regional base acreage, crop prices, and regional water allocations. SWAP diverges from the modeling techniques presented in the CVPM in three ways. The specification of the production function is more flexible and uses a different calibration technique, water is spatially and temporally allocated over smaller units; the data are extrapolated to the year 2020. SWAP uses projections of structural changes in output markets and forecasted contractions in irrigated agricultural acreage from Bulletin 160-98 to create the 2020 extrapolation.

Table A-1. Economic Model Comparison

	CVPM	SWAP
Regions	21 Central Valley	21 Central Valley & 4 Southern California
Production Costs	Quadratic PMP costs by crop & region what is a PMP cost?	Market price for inputs
Production Technology	Fixed yield per acre. CES trade-off between cost and water use	Variable yield with ME quadratic production function in land, water & cost by crop & region
Output price	Prices change with total production	Fixed price with regional differences
Water Use	Yearly	Monthly

THE STATEWIDE WATER AND AGRICULTURAL PRODUCTION MODEL

The Statewide Water and Agricultural Production Model is an economic optimization model that identifies demand for water for each region (input), along with the resulting value of agricultural output. By using a supply-demand approach, SWAP is able to impute the *shadow value* per unit of water, by region and month. This approach explicitly recognizes the effect of high user prices on water demand, and conversely the effect of the willingness to pay for water reliability. Although the model is constrained by spatial water constraints, which include physical limitations on annual water availability, the optimal solution allows for transfer of water between different months such that the marginal value of water by month and crop is equated.

A shadow value represents the “true” value of an additional unit of water to a buyer in the region. Generally speaking, this additional unit of water would in turn produce additional agricultural output, whose value is dependent upon the type of crop grown and the price that is specific to the region. Only the SWAP model explicitly recognizes each region's unique willingness-to-pay for water as a function of its productive opportunities and *adapts* to changing surface supply scenarios.

SWAP's objective is to maximize each region's total net returns from agricultural production subject to pertinent production and resource constraints on water and land. Production constraints are in the form of functional relationships describing the productive tradeoffs between land and water-use efficiency, in conjunction with capital cost expenditures. The model distributes water supply based upon each region's annual water allocation, water costs and the production opportunities facing the region. The model assumes a perfectly competitive market structure such that producers are not able to influence prices in either input or output markets; each producer is perceived as being relatively small in relation to the market. Furthermore, this model is calibrated against observed data and is consistent with microeconomic theory, which asserts that productive decisions are based upon marginal conditions, whereas published data are based upon average conditions. The divergence between the average and marginal conditions, either in the context of costs or revenues, is attributed to additional information not contained in the data, but observed in practice. Such divergences can be the result of heterogeneous land and resources quality, on-farm productive capacity, economies of scale, etc. In terms of model calibration, economic theory requires that value of the marginal product for all inputs must be the same, across all months. Using water as an example, this implies that each region's shadow value on water would be the same for every month, in a given year. Otherwise, producers (regions) could be made better off by shifting the water supply across months. In reality, regions are constrained by the quantity of water that can be held for future months usage.

This model is calibrated using a two-step process that engages both Positive Mathematical Programming (Howitt 1995) and Maximum Entropy Modeling Techniques, which have been most recently applied to an agricultural production setting by Paris and Howitt (1998).

The Empirical Setting

The Statewide Water and Agricultural Production model is a multi-input, multi-output economic optimization model that includes the original 21 CVPM regions that span the Central Valley of California. Refinements contained within the SWAP model include a spatial extension of the agricultural production regions to include 4 regions in Southern California. Descriptions of each of the regions are provided in Table A-1. In addition, the SWAP model examines agricultural water demands on a monthly basis and extrapolates the model to consider agricultural water demands for the year 2020. Irrigated acreage, by crop and region, for the base year 1995 and forecasts for the year 2020 are based upon those data presented in Bulletin 160-98. The inclusion of these refinements allows for a more robust representation of alternative water management policies that can be interpreted on a statewide basis.

Table A-2. Swap Agricultural Regions

CVPM/ SWAP Region	Description
1	CVP Users: Anderson Cottonwood, Clear Creek, Bella Vista, Sacramento River miscellaneous users.
2	CVP Users: Corning Canal, Kirkwood, Tehama, Sacramento River miscellaneous users.
3	CVP Users: Glenn Colusa ID, Provident, Princeton-Cordora, Maxwell, Colusa Basin Drain MWC, Orland-Artois WD, Colusa County, Davis, Dunnigan, Glide, Kanawha, La Grande, Westside WD, and Tehama Colusa Canal Service Area.
4	CVP Users:Princeton-Cordora-Glenn, Colusa Irrigation Co., Meridian Farm WC, Pelger Mutual WC, Reclamation Districts 1004 and 108, Roberts Ditch, Sartain M.D., Sutter MWC, Swinford Tract IC, Tisdale Irrigation, Sacramento River miscellaneous users.
5	Most Feather River riparian and appropriative users.
6	Yolo and Solano Counties, CVP users: Conaway Ranch, and Sacramento River miscellaneous users.
7	Sacramento Company north of the American River, CVP Users: Natomas Central MWC, Pleasant Grove-Verona, San Juan Suburban, Sacramento River Miscellaneous users.
8	Sacramento County south of the American River, San Joaquin Company
9	Delta Regions. CVP users: Banta Carbona, West Side, Plainview.
10	Delta Mendota Canal. CVP Users:
11	Stanislaus River water rights: Modesto ID, Oakdale ID, South San Joaquin ID.
12	Turlock ID.
13	Merced ID, CVP Users: Madera, Chowchilla, Gravelly Ford.
14	CVP Users: Westlands
15	Tulare Lake Bed. CVP Users: Fresno Slough, James, Tranquility, Traction Ranch, Laguna, Reclamation District 1606.
16	Eastern Fresno Company, CVP Users: Friant-Kern Canal, Fresno ID, Garfield, International.
17	CVP Users: Friant-Kern Canal, Hills Valley, Tri-Valley Orange Grove.
18	CVP Users: Friant-Kern Canal, County of Fresno, Lower Tule River ID, Pixley ID
19	Kern Co. SWP service area
20	CVP Users: Friant-Kern Canal, Shafter-Wasco, South San Joaquin.
21	CVP Users: Cross Valley Canal, Friant-Kern Canal, Arvin Edison
22	Imperial County
23	Palo Verde ID
24	Coachella Valley ID
25	San Diego County

Source: CVPIA, 1997, Technical Appendix Volume Eight

Regional Crops, Prices, and Production Costs

Regional crop prices are based upon data used in the CVGSM-CVPM model and were generated using annual county agricultural commissioner reports. The data used in this model includes the years 1985 to 1992, with prices normalized to the 1992 level. Data for the Southern California regions were generated using methods that are consistent with the assumptions underlying the CVPM data set. Descriptions of the crop categories are contained in Table A-2. Production costs are generated using a multi-input, multi-output production model that uses a maximum entropy calibration technique for each region and crop (Paris and Howitt, 1998). This model allows for explicit substitution among the three inputs – land, water, and capital.

Table A-3. Agricultural Crop Descriptions

Crop Category	Description of Included Crops
Northern Model:	**Regions 1-21**
Cotton	Cotton
Field	Field Corn
Fodder	Alfalfa hay, Pasture, Miscellaneous Grasses
Grain	Wheat
Grapes	Table, Raisin, and Wine Grapes
Orchard	Almonds, Walnuts, Prunes, and Peaches
Pasture	Irrigated Pasture
Tomatoes	Processing Tomatoes
Rice	Rice
Sugarbeets	Sugarbeets
Subtropical	Olives, Figs, and Pomegranates
Truck	Melons, Onions, Potatoes, and Miscellaneous Vegetables
Southern Model	**Regions 22-25**
Cotton	Cotton
Field	Field Corn and Miscellaneous Field Crops
Fodder	Alfalfa hay
Grapes	Wine
Tomatoes	Fresh Market and Processing Tomatoes
Multi-Grain	Wheat
Orchard	Dates, Walnuts, and Peaches
Pasture	Irrigated Pasture
Subtropical	Avocado
Sugarbeets	Sugarbeets
Truck	Broccoli, Cabbage, Cauliflower, Onion, Lettuce, Melon, and Potato

Source: CVPIA, 1997, Technical Appendix Volume Eight; Various Counties Agricultural Crop and Livestock Report, Various Years

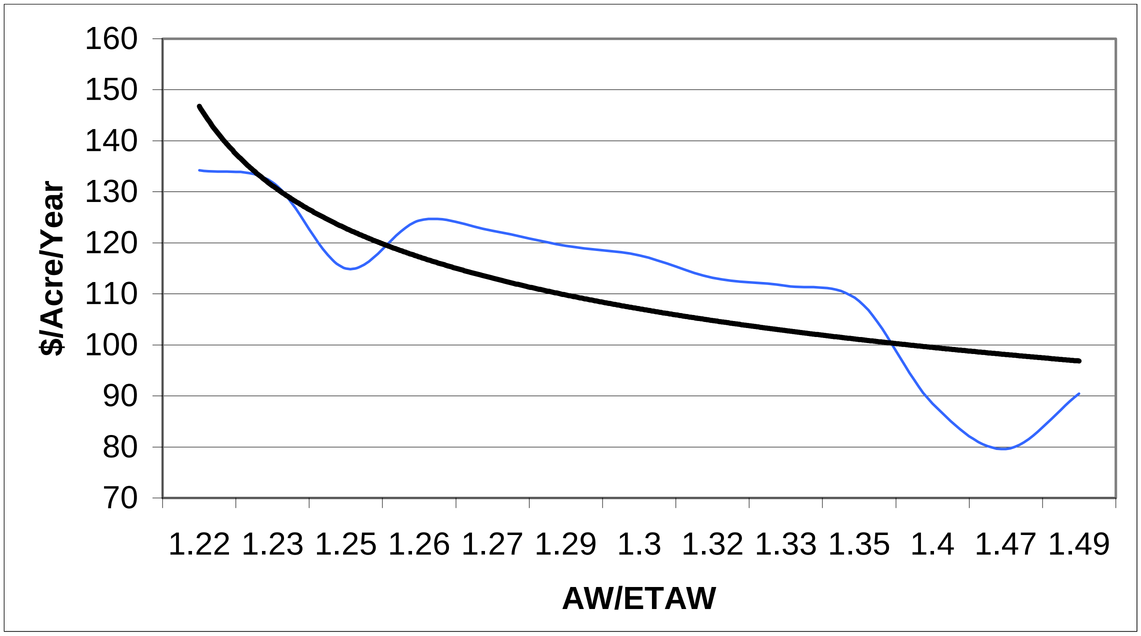
Capital Costs

The variable cost of capital represents the annual irrigation system cost for an equivalent yield that is normalized to one acre. The variable capital cost defines a functional relationship between the cost of irrigation technology and improvements in water use efficiency. Thus, investments in “better” irrigation technology result in an increase in irrigation efficiency, which can be interpreted as using less applied water to achieve the same yield. A Constant Elasticity of Substitution (CES) cost function is used to estimate the variable capital cost by using the CES parameter estimates presented in CVPM. The CES cost function is as follows:

$$(1) \quad A1_{g,j} \bullet [B1_{g,j} \bullet EFFC_{g,j}^{(\rho-1/\rho)} + (1-B1_{g,j}) \bullet CAPITL_{g,j}{}^{(\rho-1/\rho)}]^{(\rho/\rho-1)} = 1$$

Where A1 is a scale parameter and B1 is a distribution parameter for the CES function obtained from the CVPM. These estimated parameters were generated using nonlinear least squares. EFFC is the measure of water use efficiency, for each crop and region, and is obtained by taking the ratio of applied water to the Evapotranspiration of Applied Water (AW/ETAW).

Figure A-1. Illustrating the Trade-off Between Irrigation Efficiency and Cost of Capital



Source: CVPIA, 1997, Technical Appendix Volume Two

Regional Water Supplies, Water Costs, and Crop Water Requirements

Estimates of each region’s water supply and water cost were obtained from CVPM data sets of four potential sources of water: Central Valley Project (CVP), State Water Project (SWP), and Local Surface Water and Groundwater. Regional estimates for the two former supplies were obtained from the U.S. Bureau of Reclamation operations data (1993, 1994) and the Department of Water Resources Bulletin 132 (various years), respectively. Estimates of regional surface and groundwater supplies were obtained using the Central Valley Ground-Surface Water Model (CVGSM). The cost per acre-foot of water for each region uses a weighted-average of the cost of water for all districts' aggregate supply, within the region.

The annual amount of applied water that is allocated by crop and region is the solution to the production optimization problem. The annual quantity is apportioned by month, based on monthly crop water requirements, which have been identified for each crop and region using the Department of Water Resources Consumptive Use Model (1997).

Land

Regional crop acreage data were obtained from CVPM. These data were compiled using crop acreage data from County Agricultural Commissioner Reports and the Department of Water Resources 1990 Land Use Estimates. Total regional irrigated acreage was calibrated against Bulletin 160-98 for both the base year 1995 and the year 2020.

THE STATEWIDE WATER AND AGRICULTURAL PRODUCTION ECONOMIC MODEL

Crop production is modeled using a multi-input production model for each region and crop. The model captures the three ways in which farmers can adjust crop production when faced with changes in the price or availability of water. The total amount of irrigated land in production can change with water availability and price. This reaction is particularly observed during California's periodic droughts when the largest reduction in water use comes from a reduction in irrigated acres. The second avenue of adjustment is by changing the mix of crops produced so that the value produced by a unit of water is increased. This is termed the extensive margin of substitution. The third approach is termed the intensive margin of substitution and measures the changes in the intensity of input use on the crops that are grown. Since crop production is a function of land, water, and capital inputs, the intensive margin of substitution can take the form of stress irrigation or substituting capital for applied water.

Production Function Specification

Each region has a different production function for each of the crops produced. Within a region the production of different crops is connected by the restrictions on the total land and water inputs available. Crop production is modeled using a multi-input production model for each region and crop. The quadratic form of the production function is one of the simplest functional forms that will allow for decreasing marginal returns to additional input and substitutability of inputs, as required by theory. Several different agricultural inputs have been aggregated and simplified to land, water, and capital.

Since crop production is a function of land, water, and capital, substitution between these inputs can take the form of stress irrigation or substituting capital for applied water. The capital input is an amalgam of labor management and capital used to improve irrigation efficiency under different technologies. The CVPM analysis showed the trade-off between irrigation efficiency and the cost per unit of applied water in terms of the capital and management costs to apply it. In

SWAP, this characteristic is modeled more generally by allowing substitution among all three inputs in production.

The production function is written in general as:

$$(2) \qquad y = f(x_1, x_2, x_3)$$

The specific quadratic used in the SWAP model has the form:

$$(3) \quad y = \left[\alpha_1 \ , \ \alpha_2, \alpha_3 \ \right] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} - \left[x_1 \quad x_2 \quad x_3\right] \begin{bmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} \\ \gamma_{21} & \gamma_{22} & \gamma_{23} \\ \gamma_{31} & \gamma_{32} & \gamma_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

where y is the total regional output of a given crop and x_i is the quantity of land, water or capital allocated to regional crop production.

The total problem defined over G regions and i crops in each region for a single year is:

$$(4) \qquad \begin{aligned} &\text{Max} \quad \Sigma_G \, \Sigma_i \quad p_i \, f_{Gi}(x_1, x_2, x_3) - \omega_1 x_1 - \omega_2 x_2 - \omega_3 x_3 \\ &\text{subject to} \quad \Sigma_{Gi} \, x_{1Gi} \le X_1 \qquad (\text{ Land }) \\ &\qquad\qquad\quad \Sigma_{Gi} \, x_{2Gi} \le X_2 \qquad (\text{ Water }) \end{aligned}$$

where the total annual quantities of irrigated land and water (X_1 and X_2) are limited in each region and must be optimally allocated across crops grown in that region . The CALVIN analysis requires that agricultural and urban water be valued by month. This monthly valuation of water requires an extension of the usual optimization model specification.

Monthly Irrigation Water Use Specification

Water must be simultaneously allocated by month and year over the set of potential crops. The annual applied water for a given crop and region is determined by optimizing the constrained regional production problem defined above. In allocating land, water, and capital, the model equates the value of marginal product for each crop with the sum of the regional monetary cost and shadow value. Essentially, the SWAP model allows technology to be adjusted each year, but once the optimal annual choice of capital expenditure is made, the same technology is used for each month in the growing season.

The monthly pattern of water application is restricted so that the same monthly proportions of the annual applied water are applied despite changes in the total annual applied water to a crop under different levels of technology or stress.

The SWAP model assumes that water supplies can be reallocated sufficiently between months during any irrigation season to enable the farmer to optimize water use in a given year. This leads to the equality of the value of marginal water in all months, and consequently the same opportunity cost of water in all irrigation months. Without this freedom to have small changes in the delivery timing of water, the opportunity cost of water will fluctuate between months driven by small quantity differences across the months. The SWAP model was tested for the quantity of these reallocations, which were found to be less than five percent of the monthly supply in all cases for the base run. It is reasonable to assume that irrigation districts have this flexibility in their operations.

The water constraints in the model for i crops and a single region G are:

$$\begin{aligned} &Annual \quad && \sum_i x_{G,Water,i} \leq X_{G,Water} \\ &Monthly \quad && \sum_i x_{G,Water,i}\, pro_{G,M,i} = \tilde{X}_{G,M,Water} \end{aligned} \tag{5}$$

Where $x_{g,water,i}$ is the annual water allocated to crop i in region G, and $pro_{G,m,i}$ is the monthly proportion of the annual water that is applied to that crop in that region. The values of $pro_{G,m,i}$ are derived from data on the monthly regional ET requirements for the crops. At this stage of the model, ET is not adjusted to annual climate conditions as we are trying to model the expected water allocations that the farmer anticipates. By summing over the monthly proportions for different crops, the monthly regional total, $\tilde{X}_{G,m,water}$ is obtained. The model will allocate crop types and acreages in the region so that the annual net returns are maximized and the monthly opportunity cost of water is equalized across months. Despite the equality of the monthly values of water, the differences in quantity used for a given crop mix in each month ensures that the monthly demand functions, derived by parameterizing the annual water available, differ by month and region.

Calibration of the Economic Model

Calibration of the full set of parameters for the production function requires that each regional crop be parameterized in terms of nine parameters, three for the linear terms, and, under the usual symmetry assumptions, six for the quadratic matrix. In a single set of base year data the number of equations are limited to three first order conditions and one average production condition. We are faced with the unusual condition where we need to solve for more parameters (9) than the number of restrictions (4). In short, we have negative degrees of freedom. The ill-posed problem of reconstructing nine parameters from four constraints can be solved using a new method termed maximum entropy (ME) analysis. Paris & Howitt (1998) show how the ME approach can be used to calibrate cost function models to farm data. In this study, the approach is developed for crop and regional production functions.

The SWAP model is reconstructed in three stages. In the first stage, a linear programming (LP) model is constructed for each region that incorporates all the available data on cropping acreages, annual water use, yields, output prices and input costs and quantities. The model is maximized subject to land and water constraints and also a set of constraints that calibrate the model to the observed land use and production quantities in each region. This initial stage is the same as that used in the positive programming approach (Howitt 1995).

In stage two the shadow values from the LP stage, and the quadratic production function specification in equation (3) are used to specify the first order conditions for input allocation and the average product condition. The unknown parameter values are represented by defining a set of prior support values, and unknown probabilities for these values. The marginal conditions for the base period are defined in terms of the expected parameter values which can have values at any point within the support set values depending on the probabilities. The problem is solved by specifying the problem as a nonlinear constrained optimization that maximizes the entropy of the parameter probabilities subject to the marginal conditions. This method can be shown to optimize the information content of the model parameters while reproducing the base year land, water and capital allocations (Paris, Howitt 1998).

The expected value of the parameters are now calculated from the probabilities derived in the ME stage, and used in the production function for the regional model defined in equation (4). The base year model calibrates the regional land and water allocations very closely to the base data, and is used to generate the monthly water value step functions by the parameterization process described below.

PARAMETERIZATION

One of the difficulties in identifying viable water policies in California rests with policy makers' inability to predict future water supply, which is a function of stochastic weather conditions. Although the State's historical supply data are represented by average conditions, California rarely experiences *average* water supplies. Thus, it is important to be able to consider a range of potential supply scenarios. The SWAP model is parameterized, by 10% increments, from 60% of the base year's supply to 120%.

INTERACTIONS BETWEEN SWAP AND CALVIN

Monthly estimates of the economic valuation of water for 25 regions are derived using the Statewide Water and Agricultural Production. These estimated demand functions are electronically transferred to the CALVIN model where they are used to determine the statewide allocation of water supply across 72 years of variable hydrology.

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